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STANDARD MODEL DECAYS OF TAU INTO THREE CHARGED LEPTONS

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ABSTRACT

Analytic expressions are given for the τ decays into three charged leptons, $\tau \rightarrow \ell\ell\bar{\ell}\nu_\tau\bar{\nu}_\ell$, where the ℓ are combinations of electrons and muons, and for the radiative decays $\tau \rightarrow \gamma\ell\nu_\tau\bar{\nu}_\ell$. The branching ratios are sensitive functions of whether the final state ℓ are muons or electrons.

The branching ratio for the process $\mu^+ \rightarrow e^+e^+e^-\nu_e\bar{\nu}_\mu$ was first measured more than thirty years ago¹⁻⁴. The measurements are in good agreement with the standard model theoretical predictions⁵. More recently, the primary interest in this process is as a background for processes that would signal new physics beyond the standard model such as $\mu^+ \rightarrow e^+e^+e^-$.

The study of τ decays is rapidly reaching the point where similar studies could be done. At CLEO, for example, enough τ s' may have already been produced to allow measurement of the branching ratios for the four leptonic final states, $e^+e^+e^-\nu_e\bar{\nu}_\tau$, $\mu^+e^+e^-\nu_\mu\bar{\nu}_\tau$, $e^+\mu^+\mu^-\nu_e\bar{\nu}_\tau$ and $\mu^+\mu^+\mu^-\nu_\mu\bar{\nu}_\tau$; or to place limits on the branching ratios for the corresponding lepton number violating processes which do not involve neutrinos in the final states⁶. The purpose of this note is to give the standard model branching ratios for $\ell^+\ell^+\ell^-\nu_\ell\bar{\nu}_\tau$, where $\ell = e$ or μ ; and to present an analytic expression for the square of the matrix element which can be easily incorporated into Monte Carlo simulations.

The processes proceed through the Feynman diagrams given in Figure 1 where the last two diagrams are needed only for the cases of identical particles in the final state. Each term in the square of the matrix element is of the form

$$Tr \gamma^\beta (1 - \gamma_5) \gamma \cdot p_5 \gamma^\alpha (1 - \gamma_5) A \times Tr \gamma_\alpha (1 - \gamma_5) \gamma \cdot p_4 \gamma_\beta (1 - \gamma_5) B$$

where A and B are strings of gamma matrices and p_4 and p_5 are the momenta of the neutrinos. If these traces are done directly the result involves many terms. However, the result can be made shorter by first introducing an identity matrix on either side of A and of B , and Fierz rearranging both. The expression then becomes

$$\begin{aligned} & Tr A \gamma_\sigma (1 + \gamma_5) Tr \gamma_\beta (1 - \gamma_5) \gamma \cdot p_5 \gamma_\alpha (1 - \gamma_5) \gamma^\sigma \\ & \times Tr B \gamma_\rho (1 + \gamma_5) Tr \gamma^\alpha (1 - \gamma_5) \gamma \cdot p_4 \gamma^\beta (1 - \gamma_5) \gamma^\rho \\ & \sim p_5^\rho p_4^\sigma Tr A \gamma_\sigma (1 + \gamma_5) Tr B \gamma_\rho (1 + \gamma_5) \end{aligned}$$

and the squared matrix element is greatly simplified. Further, since this is the only dependence on p_4 and p_5 , the phase space integrals of the neutrinos can be easily done.

The total rate for the process $\tau^+(P) \rightarrow \mu^+(p_1) + e^+(p_2) + e^-(p_3) + \bar{\nu}_\tau + \nu_\mu$ is given by

$$\Gamma = \left[\frac{G}{\sqrt{2}} \right]^2 \frac{\alpha^2}{(2\pi)^8} \frac{128}{3} \frac{1}{M} \int \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} \frac{d^3 p_3}{2E_3} \Theta(Q^2) \Theta(Q^0) \frac{1}{s^2} T \quad (1)$$

where $Q^\mu = P^\mu - p_1^\mu - p_2^\mu - p_3^\mu$, $P^2 = M^2$, $p_1^2 = m_\mu^2$, $p_2^2 = m_e^2 = p_3^2$, and $s = (p_2 + p_3)^2$. The Θ functions are what remain after analytically computing the integrals over the neutrino momenta. The rest of the squared matrix element, T , is given in Appendix I. Expressions for the rate to the other final states can be obtained by interchanging m_μ and m_e or by setting them equal. In addition, for the identical lepton cases, the expression for $\frac{1}{s^2}T$ has to be generalized as explained in the appendix. The rate for $\mu^+ \rightarrow e^+e^+e^-\nu_e\bar{\nu}_\mu$ can also be obtained from (1) by setting m_μ to m_e and then M equal to m_μ .

The branching ratios are given by the following table

$\mu^+ \rightarrow e^+e^+e^-\nu_e\bar{\nu}_\mu$	$3.60 \pm 0.02 \times 10^{-5}$
$\tau^+ \rightarrow e^+e^+e^-\nu_e\bar{\nu}_\tau$	$4.15 \pm 0.06 \times 10^{-5}$
$\tau^+ \rightarrow \mu^+e^+e^-\nu_\mu\bar{\nu}_\tau$	$1.97 \pm 0.02 \times 10^{-5}$
$\tau^+ \rightarrow e^+\mu^+\mu^-\nu_e\bar{\nu}_\tau$	$1.257 \pm 0.003 \times 10^{-7}$
$\tau^+ \rightarrow \mu^+\mu^+\mu^-\nu_\mu\bar{\nu}_\tau$	$1.190 \pm 0.002 \times 10^{-7}$

where the errors are due to inaccuracies in the numerical integration. These numbers assume the branching ratio for $\tau \rightarrow e\nu_\tau\nu_e$ is 0.177.

For the tau cases where the pair produced are electrons, the branching ratio is of the same order as the muon decay. If the pair produced are muons, however, the branching ratio is two orders of magnitude smaller. This seems to be just the effect of the $1/s^2$ in Eq. (1); for electrons s can be a lot smaller than for muons. ($s \geq 4m^2$ where m is the electron or muon mass.) This means that any experimental cut on the minimum value of invariant mass of the produced pair will drastically reduce the number of events. This also means that the cross terms between the first and second diagrams of Fig. 1 and the third and fourth diagrams make only a small contribution to the rate because these cross terms go as $1/ss'$, where $s' = (p_1 + p_3)^2$, and s and s' cannot be small simultaneously. These observations were verified by our numerical results.

An analysis of the angular distributions of the final state leptons reveals strong correlations. For the case with three electrons in the final state we have plotted, in Fig. 2a, the distributions in the angles between the two like-sign and the opposite-sign electrons. In both cases the distributions peak when the two momenta are parallel. Similar results are obtained for the

final state $\mu^+e^+e^-\nu_\mu\bar{\nu}_\tau$, but the correlations disappear for the final states $e^+\mu^+\mu^-\nu_e\bar{\nu}_\tau$ and $\mu^+\mu^+\mu^-\nu_\mu\bar{\nu}_\tau$. Again these results are simply a consequence of the $1/s^2$ term dominating the dynamics. In Fig. 2b we have plotted the distributions in the total energy E_c of the final state charged leptons. For comparison we also display the same distribution for just the phase space without any dynamics. The dynamics shift the E_c distribution to lower values. The shift to lower values increases with decreasing mass of the final state leptons.

For completeness we also include an expression for the radiative decay $\tau(P) \rightarrow \ell(p_2) + \gamma(k) + \nu_\tau + \nu_\ell$ for $\ell = e$ or μ . With the neutrinos again integrated out the rate is given by

$$\Gamma = \frac{G^2}{(2\pi)^2} \frac{4}{3} \frac{1}{M} \int \frac{d^3k}{2\omega} \frac{d^3p_2}{2E_2} \Theta(K^2) \Theta(K^0) T_\gamma \quad (2)$$

where $K^\mu = P^\mu - k^\mu - p_2^\mu$ and T_γ is given in Appendix II. The photon must be visible and thus the photon energy has some nonzero lower limit ω_0 . This form would seem to be useful if further experimental cuts need to be applied to the photon or charged lepton. If, however, no further cuts are required, then the integrals can be done analytically and the branching ratio to $\tau \rightarrow \ell + \nu_\tau + \nu_\ell$ is given by⁷

$$\begin{aligned} R = & \frac{\alpha}{3\pi} \left\{ \left(\ell n \frac{M}{m} - \frac{17}{12} \right) \left[6 \ell n \frac{1}{y_0} - 6(1-y_0) - (1-y_0)^4 \right] \right. \\ & + 3 [L(1) - L(y_0)] - \frac{1}{2} [6 + (1-y_0)^3] (1-y_0) \ell n(1-y_0) \\ & \left. + \frac{1}{48} (1-y_0)(125 + 45y_0 - 33y_0^2 + 7y_0^3) \right\}, \quad (3) \end{aligned}$$

where $y_0 = \omega_0/\omega_{\text{Max}}$ and L is a Spence function defined with an overall sign such that $L(1) = -\pi^2/6$. A graph of the branching ratio given by this equation is shown in Fig. 3 for the three processes $\mu \rightarrow e + \gamma + \nu + \bar{\nu}$, $\tau \rightarrow e + \gamma + \nu + \bar{\nu}$, and $\tau \rightarrow \mu + \gamma + \nu + \bar{\nu}$.

In conclusion we have found that with about one million τ s the standard model branching ratio for τ decays to the final states $e^+e^+e^-\nu_e\bar{\nu}_\tau$ and $\mu^+e^+e^-\nu_\mu\bar{\nu}_\tau$ may be measurable. The distributions in the angles between the final state leptons are strongly peaked at zero and stringent angular separation cuts can drastically reduce the rates.

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APPENDIX I

For the decay $\tau^+(P) \rightarrow \mu^+(p_1) + e^+(p_2) + e^-(p_3) + \bar{\nu}_\tau + \nu_\mu$ define

$$\begin{aligned}
Q^\mu &= P^\mu - p_1^\mu - p_2^\mu - p_3^\mu \\
s &= (p_2 + p_3)^2 \\
s' &= (p_1 + p_3)^2 \\
t &= p_1 \cdot p_2 \\
u &= Q \cdot P \\
s_1 &= P \cdot p_1 \\
s_2 &= P \cdot p_2 \\
s_3 &= P \cdot p_3 \\
u_1 &= Q \cdot p_1 \\
u_2 &= Q \cdot p_2 \\
u_3 &= Q \cdot p_3
\end{aligned}$$

Obviously there are relations among these invariants but the following analytic expression is simpler if we use an overcomplete set. In terms of these the expression for T in Eq. (1) is given by

$$\begin{aligned}
T &= \frac{1}{s - 2(s_2 + s_3)} \left[- (Q^2 s_1 + 2uu_1)(m_e^2 + s_2 + s_3) \right. \\
&+ \frac{1}{2} Q^2 (2s_2 t + s_3 (s' - m_e^2 - m_\mu^2)) + 2u_1(u_2 s_2 + u_3 s_3) \Big] \\
&- \frac{2}{[s - 2(s_2 + s_3)]^2} \left[\frac{1}{2} Q^2 (2s_1 - 2t - s' + m_e^2 + m_\mu^2) + 2u_1(u - u_2 - u_3) \right] \\
&\times \left[s_5^2 + s_4^2 + \frac{1}{2} s M^2 + m_e^2 (s_2 + s_3) \right] \\
&+ \frac{1}{s + s' + 2t - m_e^2 - m_\mu^2} \left[(Q^2 s_1 + 2uu_1) \left(t + \frac{s'}{2} - m_e^2 - \frac{m_\mu^2}{2} \right) + t(Q^2 s_2 + 2uu_2) \right. \\
&\quad \left. + (Q^2 s_3 + 2uu_3) \frac{1}{2} (s' - m_e^2 - m_\mu^2) \right] \\
&+ \frac{2}{[s + s' + 2t - m_e^2 - m_\mu^2]^2} \left[Q^2 (s_1 + s_2 + s_3) + 2u(u_1 + u_2 + u_3) \right] \\
&\times \left[-t^2 - \frac{1}{4} (s' - m_e^2 - m_\mu^2)^2 - \frac{1}{2} s m_\mu^2 + \frac{1}{2} m_e^2 (2t + s' - m_e^2 - m_\mu^2) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{s - 2(s_2 + s_3)} \frac{1}{s + s' + 2t - m_e^2 - m_\mu^2} \\
& \times \left[-s(s_2 + s_3)(Q^2 m_\mu^2 + 2u_2^2) \right. \\
& + (Q^2 t + 2u_1 u_2) (ss_1 + s_3 (s' - 2t - s + m_e^2 - m_\mu^2) + 2m_e^2 s_2) \\
& + \frac{1}{2} (Q^2 (s' - m_e^2 - m_\mu^2) + 4u_1 u_3) \\
& \quad \times (ss_1 + s_2 (2t - s' - s + m_\mu^2 + 3m_e^2) + 2m_e^2 s_3) \\
& + 2 (Q^2 s_1 + 2uu_1) (2ts_3 + s_2 (s' - m_e^2 - m_\mu^2) - s (s_1 + m_e^2)) \\
& + 2 (Q^2 m_e^2 + 2u_4^2) (s_3 (s' - m_e^2 - m_\mu^2) - m_e^2 s_1) \\
& + 2 (Q^2 m_e^2 + 2u_5^2) (2ts_2 - m_e^2 s_1) \\
& + \left(Q^2 \frac{s}{2} - Q^2 m_e^2 + 2u_2 u_3 \right) (2ss_1 - 4m_e^2 s_1 - 4ts_3 - 2s_2 (s' - m_e^2 - m_\mu^2)) \\
& + (Q^2 s_2 + 2uu_2) \left((s_2 - s_3) (s' - m_e^2 - m_\mu^2) + 2m_e^2 t - ss_1 + \frac{1}{2} (2m_e^2 - s) (s' - m_e^2 - m_\mu^2) \right) \\
& + (Q^2 s_3 + 2uu_3) (2t (s_3 - s_2 + 2m_e^2) - s (s_1 + t) + m_e^2 (s' - m_e^2 - m_\mu^2)) \\
& + \frac{1}{2} s (Q^2 M^2 + 2u^2) (2t + s' - m_e^2 - m_\mu^2) \\
& \left. + 6Q^2 \left(-m_e^2 (s_2 + s_3) (2t + s' - m_e^2 - m_\mu^2) + ss_3 t + 2m_e^2 ss_1 + \frac{1}{2} ss_2 (s' - m_e^2 - m_\mu^2) \right) \right].
\end{aligned}$$

If all final leptons are the same flavor set $m_\mu^2 = m_e^2$ and replace $\frac{1}{s^2}T$ by

$$\frac{1}{2} \frac{1}{s^2} T + \frac{1}{2} \frac{1}{s'^2} T (s_1 \leftrightarrow s_2, u_1 \leftrightarrow u_2, s \leftrightarrow s') + \frac{1}{2} \frac{1}{s} \frac{1}{s'} T'$$

where T' is given by

$$\begin{aligned}
& - \frac{1}{s - 2(s_2 + s_3)} \frac{1}{s' - 2(s_1 + s_3)} \\
& \times \left[(Q^2 t + 2u_1 u_2) (4s_3^2 - 2m_e^2 M^2) \right. \\
& + \left(Q^2 \left(\frac{s'}{2} - m_e^2 \right) + 2u_1 u_3 \right) (-2s_2 s_3 + s (s_3 + M^2) - m_e^2 (2M^2 + s_2 + s_3)) \\
& \left. + (Q^2 s_1 + 2uu_1) \left(2s_2 s_3 - ss_3 - \frac{1}{2} M^2 s + 2m_e^2 (s_2 + s_3) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \left(Q^2 \left(\frac{s}{2} - m_e^2 \right) + 2u_2u_3 \right) \left(-2s_1s_3 + s's_3 + s'M^2 - m_e^2 (2M^2 + s_1 + s_3) \right) \\
& + \left(Q^2s_2 + 2uu_2 \right) \left(2s_1s_3 - s's_3 - \frac{1}{2} M^2s' + 2m_e^2 (s_1 + s_3) \right) \\
& + \left(Q^2m_e^2 + 2u_3^2 \right) \left(-2t (s_3 + M^2) - m_e^2 (s_1 + s_2) \right) \\
& + \left(Q^2s_3 + 2uu_3 \right) \left(t (M^2 + 4s_3) + m_e^2 \left(2s_2 + 2s_1 + t + \frac{1}{2}s + \frac{1}{2}s' - M^2 - m_e^2 \right) \right) \\
& + \left(Q^2M^2 + 2u^2 \right) \left(-2ts_3 + m_e^2 (2s_3 - 2t - s - s' + 2m_e^2) \right) \\
& + 6Q^2 \left(-2ts_5^2 + m_e^2 \left(-2s_2s_3 - 2s_1s_3 - 2s_1s_2 + \frac{1}{2}s' (s_2 + s_3) + \frac{1}{2}s (s_1 + s_3) \right. \right. \\
& \quad \left. \left. + M^2 (2t + s' + s - 2m_e^2) \right) \right) \Big] \\
& - \frac{1}{[s + s' + 2t - 2m_e^2]^2} \\
& \quad \times \left[\left(Q^2(s_1 + s_2) + 2u(u_1 + u_2) \right) \left(-2t(s + s') + 2m_e^2 (s + s' - 2m_e^2) \right) \right. \\
& \quad \left. + 4 \left(Q^2s_3 + 2uu_3 \right) t (t - 2m_e^2) \right] \\
& - \frac{1}{s + s' + 2t - 2m_e^2} \frac{1}{s - 2(s_2 + s_3)} \\
& \quad \times \left[\left(Q^2m_e^2 + 2u_2^2 \right) ss_3 + \left(Q^2m_e^2 + 2u_4^2 \right) \left(-s's_3 + m_e^2 (s_1 + s_3) \right) \right. \\
& + \left(Q^2m_e^2 + 2u_5^2 \right) \left(-2ts_2 + m_e^2 (s_1 + s_2) \right) \\
& - \left(Q^2M^2 + 2u^2 \right) st \\
& + \left(Q^2t + 2u_1u_2 \right) \left(s_3 (s - s') - m_e^2 (s_2 + 3s_3) \right) \\
& + \left(Q^2 \left(\frac{s'}{2} - m_e^2 \right) + 2u_1u_3 \right) \left(s's_2 - ss_1 + m_e^2 (s_2 - s_3) \right) \\
& + \left(Q^2s_1 + 2uu_1 \right) \left(-2ts_3 - s's_2 + ss_1 + m_e^2 (s + 4s_3 - 2s_2) \right) \\
& + \left(Q^2 \left(\frac{s}{2} - m_e^2 \right) + 2u_2u_3 \right) \left(2ts_3 + s's_2 - ss_1 + m_e^2 (2s_1 - s_2 - s_3) \right) \\
& + \left(Q^2s_2 + 2uu_2 \right) \left(ss_1 - s's_2 + m_e^2 \left(2s_3 - \frac{1}{2}s - \frac{1}{2}s' - t + m_e^2 \right) \right) \\
& + \left(Q^2s_3 + 2uu_3 \right) \left(t(s + 2s_2) + m_e^2 (2s_3 - 2s_2 + \frac{1}{2}s - \frac{1}{2}s' - t + m_e^2) \right)
\end{aligned}$$

$$\begin{aligned}
& + 6Q^2 \left(-sts_3 + m_e^2 \left(t(s_2 + s_3) + \frac{1}{2}s'(s_2 + s_3) + \frac{1}{2}s(s_2 - s_3 - 2s_1) \right) \right. \\
& \quad \left. - m_e^2(s_2 + s_3) \right) \Big] \\
& - \frac{1}{s + s' + 2t - 2m_e^2} \frac{1}{s' - 2(s_1 + s_3)} \\
& \times \left[\left(Q^2 m_e^2 + 2u_2^2 \right) \left(-ss_3 + m_e^2(s_2 + s_3) \right) + \left(Q^2 m_e^2 + 2u_4^2 \right) s' s_3 \right. \\
& + \left(Q^2 m_e^2 + 2u_5^2 \right) \left(-2ts_1 + m_e^2(s_1 + s_2) \right) - \left(Q^2 M^2 + 2u^2 \right) s't \\
& + \left(Q^2 t + 2u_1 u_2 \right) \left(s_3(s' - s) - m_e^2(s_1 + 3s_3) \right) \\
& + \left(Q^2 \left(\frac{s'}{2} - m_e^2 \right) + 2u_1 u_3 \right) \left(2ts_3 - s's_2 + ss_1 + m_e^2(2s_2 - s_1 - s_3) \right) \\
& + \left(Q^2 s_1 + 2uu_1 \right) \left(s's_2 - ss_1 + m_e^2(2s_3 - \frac{1}{2}s - \frac{1}{2}s' - t + m_e^2) \right) \\
& + \left(Q^2 \left(\frac{s}{2} - m_e^2 \right) + 2u_2 u_3 \right) \left(ss_1 - s's_2 + m_e^2(s_1 - s_3) \right) \\
& + \left(Q^2 s_2 + 2uu_2 \right) \left(-2ts_3 + s's_2 - ss_1 + m_e^2(s' + 4s_3 - 2s_1) \right) \\
& + \left(Q^2 s_3 + 2uu_3 \right) \left(t(s' + 2s_1) + m_e^2(-t + 2s_3 - 2s_1 + \frac{1}{2}s' - \frac{1}{2}s + m_e^2) \right) \\
& + 6Q^2 \left(-s'ts_3 + m_e^2(t(s_1 + s_3) + \frac{1}{2}s'(s_1 - 2s_2 - s_3) + \frac{1}{2}s(s_1 + s_3)) \right. \\
& \quad \left. + m_e^4(-s_1 - s_3) \right) \Big]
\end{aligned}$$

APPENDIX II

The decay $\tau(P) \rightarrow \ell(p_2) + \gamma(k) + \nu + \bar{\nu}$ is given in the text in terms of T_γ . If the energy of the charged lepton, the energy of the photon, the mass of τ , and the mass of ℓ are denoted by E_2, ω, M , and m , and $k \cdot p_2$ is named u , then T_γ is given by

$$\begin{aligned}
T_\gamma &= 4u^2 \left(\frac{1}{M\omega} + \frac{1}{\omega^2} \right) \\
& + u \left[\frac{1}{\omega} \left(-5M + 3\frac{m^2}{M} - 12E_2 \right) + \frac{1}{\omega^2} \left(3M^2 - 8ME_2 + 3m^2 \right) - 4 \right] \\
& + \frac{1}{u} \left[-4\omega^2 M^2 + \omega \left(3M^3 - 12M^2 E_2 - 5Mm^2 \right) \right. \\
& \quad \left. + \frac{1}{\omega} \left(6M^3 E_2^2 - 4M^2 m^2 E_2 - 8M^2 E_2^3 + 6Mm^2 E_2^2 \right) \right]
\end{aligned}$$

$$\begin{aligned}
& +6M^3E_2 + 3M^2m^2 - 16M^2E_2^2 - 2Mm^2E_2 + 3m^4 \Big] \\
& + \frac{1}{u^2} \Big[4M^2m^2\omega^2 + \omega \Big(-3M^3m^2 + 8M^2m^2E_2 - 3Mm^4 \Big) \\
& \quad -3M^3m^2E_2 + 2M^2m^4 + 4M^2m^2E_2^2 - 3Mm^4E_2 \Big] \\
& + 4M\omega + 2M^2 + 16ME_2 + 2m^2 \\
& + \frac{1}{\omega} \Big[-3M^3 + 2M^2E_2 - 3Mm^2 + 16ME_2^2 - 6m^2E_2 \Big] \\
& + \frac{1}{\omega^2} \Big[-3M^3E_2 + 2M^2m^2 + 4M^2E_2^2 - 3Mm^2E_2 \Big]
\end{aligned}$$

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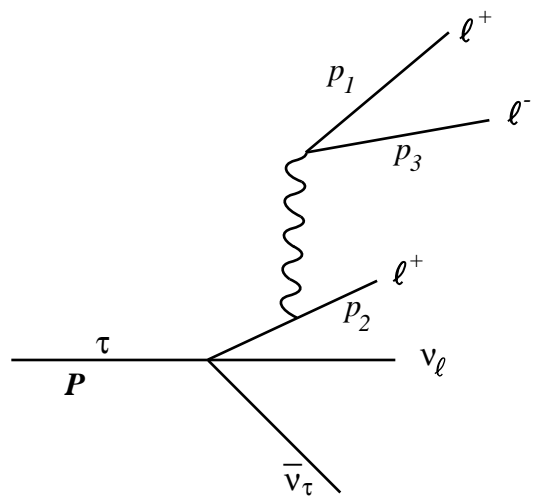
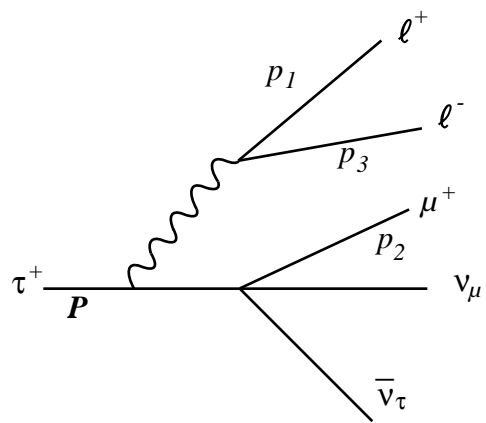
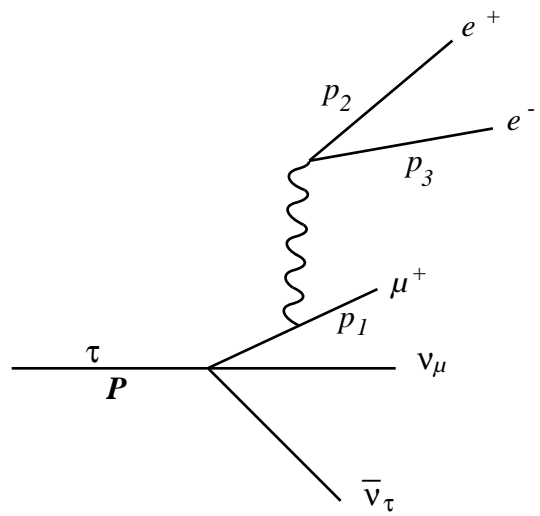
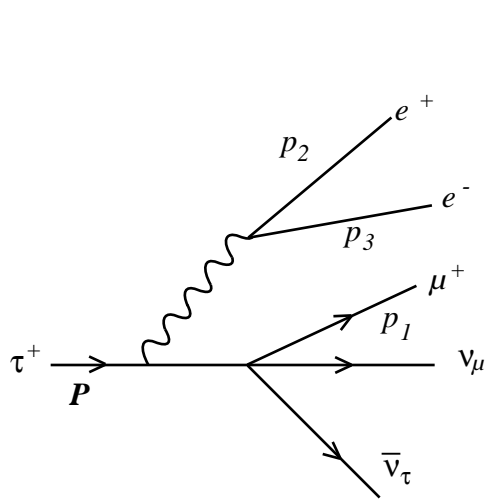
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FIGURE CAPTIONS

1. The Feynman diagrams for the decays $\tau^+ \rightarrow \ell^+ \ell^+ \ell^- \nu_\ell \bar{\nu}_\tau$ where the ℓ are electrons or muons.
2. (a) The distribution in $z_{12} = \hat{p}_1 \cdot \hat{p}_2$ for the two-like sign electrons (dashed line) and in $z_{13} = \hat{p}_1 \cdot \hat{p}_3$ for the two-opposite sign leptons (solid line). For comparison the corresponding distribution for the phase space is also plotted (dash-dot line).
 (b) The distributions in the total energy $E_c = (E_1 + E_2 + E_3)/m_\tau$ (solid line), and the corresponding distribution for the phase space (dashed line).
3. The branching ratio, R , for $\mu \rightarrow e + \gamma + \nu + \bar{\nu}$, $\tau \rightarrow e + \gamma + \nu + \bar{\nu}$ and $\tau \rightarrow \mu + \gamma + \nu + \bar{\nu}$ as a function of the minimum detected photon energy, ω_0 . $y_0 = \omega_0/\omega_{\text{Max}}$. Note that R is defined relative to the three body process $\mu \rightarrow e + \nu + \bar{\nu}$, $\tau \rightarrow e + \nu + \bar{\nu}$ or $\tau \rightarrow \mu + \nu + \bar{\nu}$ and thus, for the τ decays, is not the absolute branching ratio.

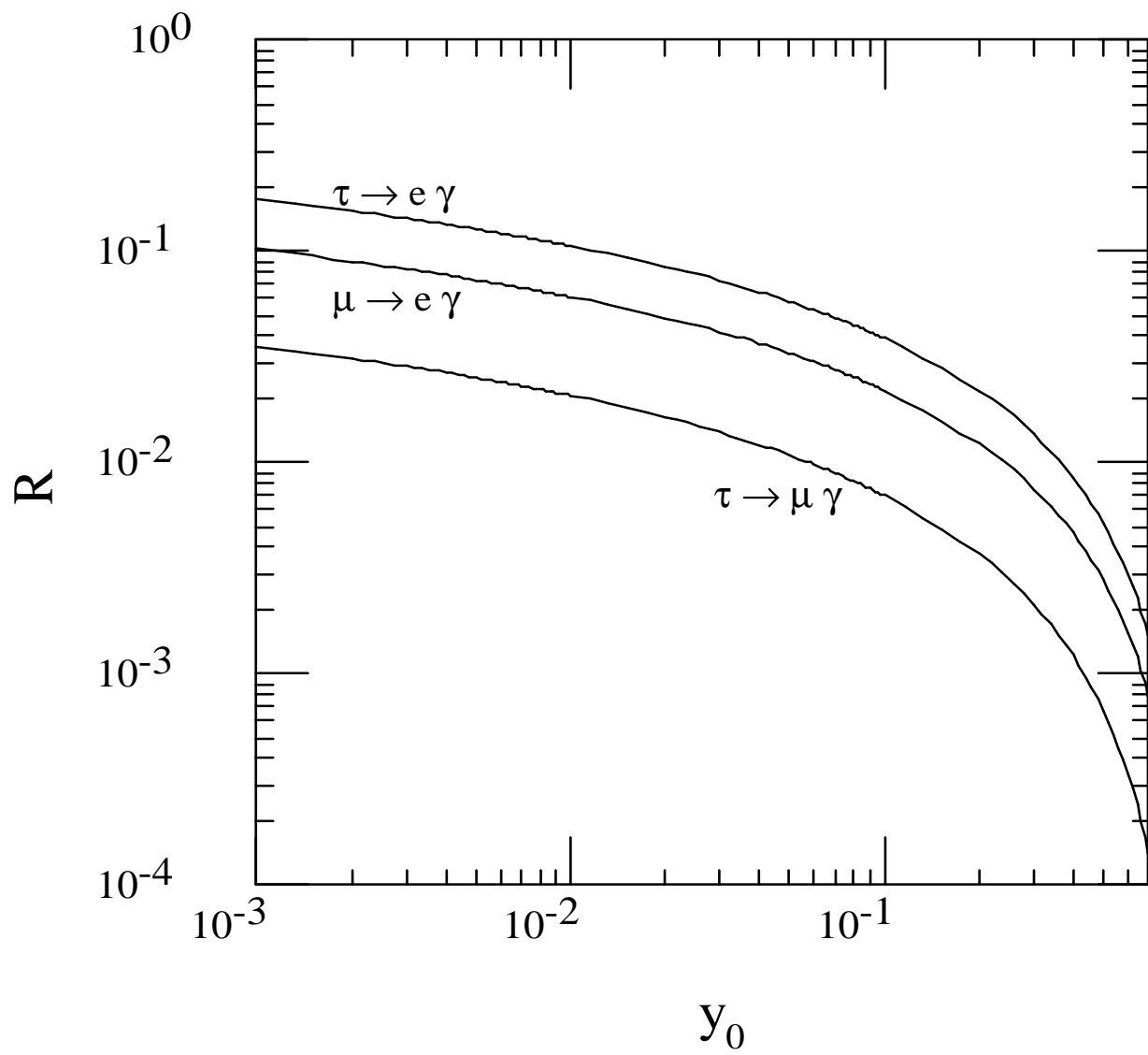
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